Homework 7 PS405

Due: Tuesday, October 11, 2016

1. Calculate the internal magnetic field ($B_{internal}$) interacting with the electron in the ground state of a hydrogen atom as the proton appears to be orbiting the electron. Assume Bohr orbits for this calculation.

$B_{internal}$ = _____ tesla

2. The spin-orbit interaction is due to the $-\vec{\mu} \cdot \vec{B}$ correction to the potential energy in the Hamiltonian. If $\vec{B}$ defines the z-direction, calculate the spin magnetic dipole moment along the z-direction. Assume $g_s = 2$.

$(\mu_e)_z$ = _____ MeV/T

3. Calculate the shift in the ground state energy of a hydrogen atom when considering the difference between the $|n\ \ell\ m_\ell\ m_s\rangle$ eigenstates (i.e., no perturbation) and the $|n\ \ell\ j\ m_j\rangle$ eigenstates (i.e., with the $\vec{S} \cdot \vec{L}$ perturbation).

ΔE = _____ eV

4. Calculate the *fixed* angle between the spin vector $\vec{S}$ and the orbital vector $\vec{L}$ in spin-orbit coupling for both $j = {}^3/_2$ state.

$\theta_{P_{3/2}}$ = _____ degrees

$\theta_{D_{3/2}}$ = _____ degrees

5. Choose a 3D state of the hydrogen atom and carry out the explicit integration required to obtain the expectation value of $1/r^3$. Compare your answer with the result obtained from the general formula $\langle \frac{1}{r^3} \rangle = \frac{2}{a^3 n^3 \ell(\ell+1)(2\ell+1)}$.

6. Deuterium is an isotope of hydrogen with one electron bound to a nucleus (the deuteron) comprising a proton and a neutron. The deuteron has spin $I = 1$, and has a gyromagnetic ratio $g_D = 0.857$, which is the only change needed to calculate the hyperfine structure of the ground state of deuterium. In the ground state, the total angular momentum $\vec{J}$ is just $\vec{J} = \vec{I} + \vec{S}$ where $\vec{S}$ is the spin of the electron.

   a. Calculate the splitting of the ground state (ΔE) and produce a figure showing the splitting between the $J=1/2$ state and the $J=3/2$ state.

   $\Delta E =$ _____ *eV*

   b. Calculate the wavelength of the emitted photon when the electron transitions from the $J=3/2$ state to the $J=1/2$ state.

   $\lambda =$ _____ *cm*

   c. What is the mean lifetime of this state?

   $\tau =$ _____ *million years*

7. The D doublet lines in a sodium atom have wavelengths of 589.0 nm and 589.6 nm. What is the magnetic field induced by the electron orbital motion?

   $B =$ _____ *tesla*